



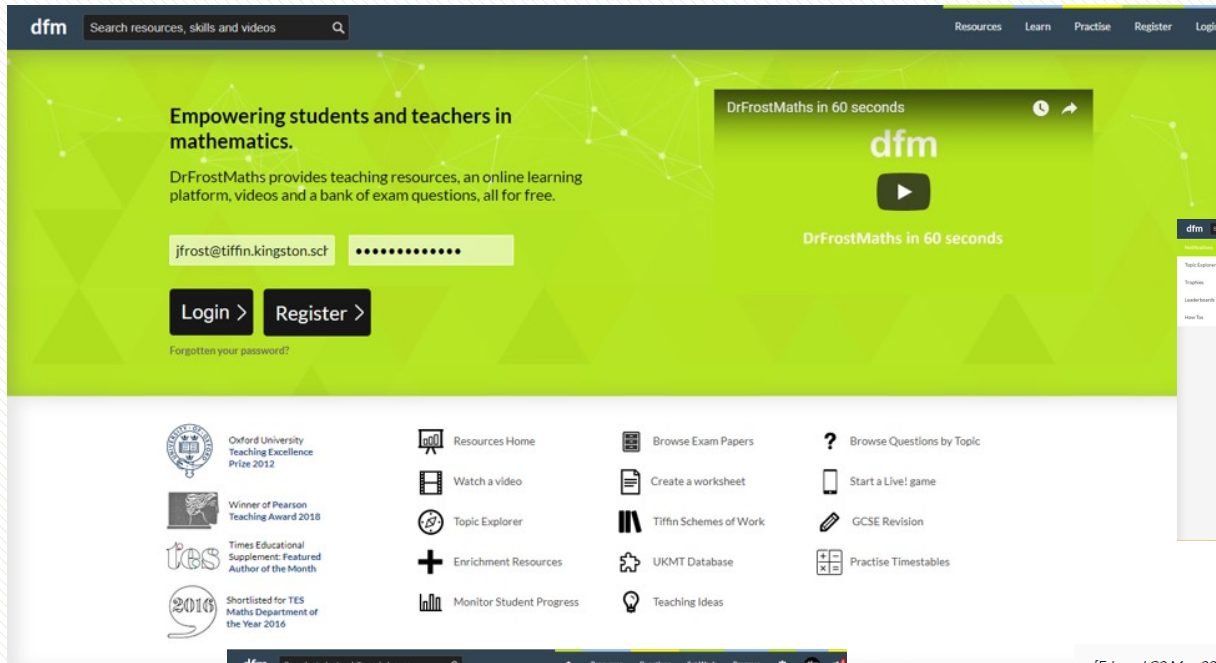
P1 Chapter 7 :: Algebraic Methods

jfrost@tiffin.kingston.sch.uk
www.dr frostmaths.com
@DrFrostMaths

www.drfrstmths.com

Everything is **completely free**.
Why not register?

Register now to interactively practise questions on this topic, including past paper questions and extension questions (including MAT + UKMT).
Teachers: you can create student accounts (or students can register themselves), to set work, monitor progress and even create worksheets



The homepage features a green header with a search bar and navigation links: Resources, Learn, Practise, Register, Login. The main content area has a large green banner with the text "Empowering students and teachers in mathematics." and "DrFrostMaths provides teaching resources, an online learning platform, videos and a bank of exam questions, all for free." Below this is a login field with the email "jfrost@tiffin.kingston.sch" and a password field. There are "Login" and "Register" buttons, and a link for "Forgotten your password?". A central video player shows "DrFrostMaths in 60 seconds" with a play button. The footer contains various accolades and a grid of links: Resources Home, Watch a video, Topic Explorer, Enrichment Resources, Monitor Student Progress, Browse Exam Papers, Create a worksheet, Tiffin Schemes of Work, UKMT Database, Teaching Ideas, Browse Questions by Topic, Start a Live! game, GCSE Revision, and Practise Timestables.

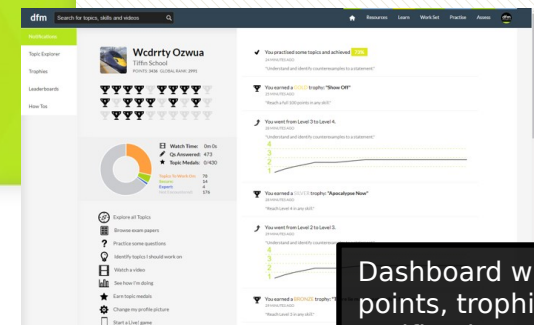
With questions by:

edexcel

OCR
Oxford Cambridge and RSA

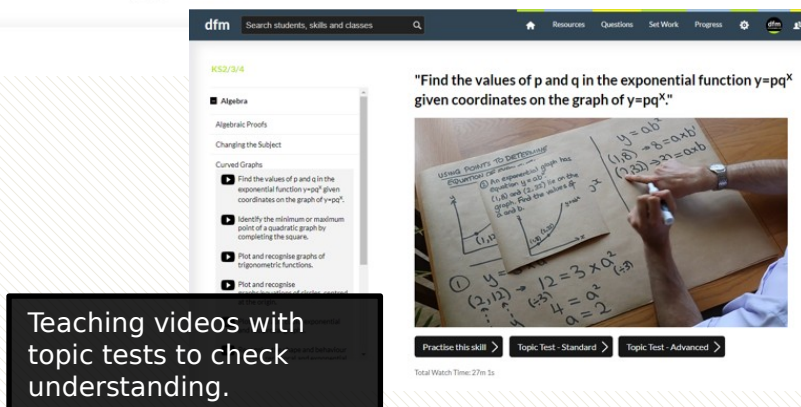
AQA

UKMT
UKMT



The dashboard shows a user profile for "Wcdrry Ozwia" with a "Watch Time" of 473 and a "Quiz Score" of 473. It includes a "Watch Time" bar chart and a "Quiz Score" bar chart. There are also "Practise" and "Watch" buttons. A sidebar on the left lists "Resources", "Learn", "Practise", and "Watch".

Dashboards with points, trophies, notifications and student progress.



The interface shows a video player with a "Practise this skill" button. The video content includes a hand-drawn graph of an exponential function $y = pq^x$ and a table of values. The text on the screen says: "Find the values of p and q in the exponential function $y = pq^x$ given coordinates on the graph of $y = pq^x$ ". The table shows: $(1, 1)$, $(2, 3)$, $(3, 9)$, $(4, 27)$, $(5, 81)$, $(6, 243)$, $(7, 729)$, $(8, 2187)$, $(9, 6561)$, $(10, 19683)$. The video also shows a hand-drawn graph of a quadratic function $y = x^2$ and a table of values: $(1, 1)$, $(2, 4)$, $(3, 9)$, $(4, 16)$, $(5, 25)$, $(6, 36)$, $(7, 49)$, $(8, 64)$, $(9, 81)$, $(10, 100)$.

Teaching videos with topic tests to check understanding.

[Edexcel C2 May 2012 Q2]

Find the values of x such that

$$2 \log_3 x - \log_3(x - 2) = 2$$

$x =$

or $x =$

Submit Answer

Questions organised by topic, difficulty and past paper.

Chapter Overview

This chapter covers a number of algebraic techniques, but also algebraic ‘proofs’.

1:: Algebraic Fractions

2:: Dividing Polynomials

Divide

3:: The Factor Theorem

Given that $x - a$ is a factor of $P(x)$, find the value of $P(a)$.

A Level 2017 specification note: The ‘Remainder Theorem’ has been removed (it was included in C2). Sad times...

4:: Proof

Prove that all square numbers are either a multiple of 4 or 1 more than a multiple of 4.

1 :: Simplifying Algebraic Fractions

Recall that you can simplify fractions by **dividing** the numerator and denominator by a **common factor**.

Fro Hint: To identify common factors we need to factorise first.

$$\frac{x^2 - 1}{x^2 + x} =$$

$$\boxed{?}$$

$$\frac{x^2 + 3x + 2}{x + 1} =$$

$$\boxed{?}$$

Fro Note: Do not leave 1 in the denominator!

2

$$\frac{2x^2 + 11x + 12}{x^2 + 9x + 20} =$$

$$\boxed{?}$$

Fro Tip: Factorise the easier one first because it provides clues to the other.

$$\frac{4 - x^2}{x^2 + 2x - 8} =$$

$$\boxed{?}$$

2
4

Fro Tip:

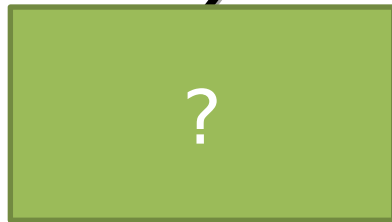
Exercise 7A

Pearson Pure Mathematics Year 1/AS

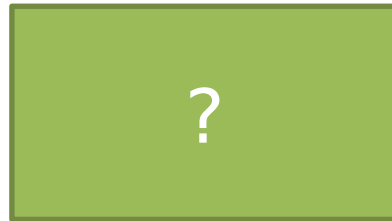
Pages 138-139

Terminology

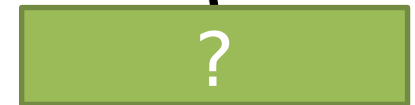
$$11 \div 4 = 2 \text{ } \textit{rem} 3$$



(the thing we're
dividing)



t



er

Normal Long Division

$$\begin{array}{r} 38. \\ 11 \overline{) 423.00} \\ \underline{33} \\ 9 \\ \underline{88} \\ 5 \\ \end{array}$$

The diagram illustrates the long division of 423.00 by 11. The divisor 11 is on the left, and the dividend 423.00 is on the right. The quotient 38. is written above the dividend. The process is shown with arrows and numbered boxes: 1. An arrow points from the first box to the '3' in the quotient, indicating the first division step. 2. An arrow points from the second box to the '30' in the dividend, indicating the multiplication of the quotient by the divisor. 3. An arrow points from the third box to the '9' in the dividend, indicating the subtraction of the product from the dividend. 4. An arrow points from the fourth box to the '5' in the dividend, indicating the bringing down of the next digit.

1. We found how many whole number of times (i.e. the quotient) the divisor went into the dividend.

2. We multiplied the quotient by the dividend.

3. ...in order to find the remainder.

4. Find we 'brought down' the next number.

How many times does 6 go into 28?

Now repeat! How many times does 3 go into 15?

$$\begin{array}{r}
 6^2 - 2 + 3 \\
 + 5 \overline{) 6^3 + 28^2 - 7 + 15}
 \end{array}$$

Multiply by 6. The first term should match with above.

$$\begin{array}{r}
 6^3 + 30^2 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 - 2^2 - 7 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 - 2^2 - 10 \\
 \hline
 \end{array}$$

Subtract and carry down next term.

$$\begin{array}{r}
 3 + \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 35 + \\
 \hline
 \end{array}$$

This is the remainder

$$\begin{array}{r}
 15 - 0 \\
 \hline
 \end{array}$$

Fro Tip:

You can check your solution by expanding:

But if you know you should get no remainder, ending with 0 at the bottom is a good

Fro Tip:

Be very careful subtracting negatives.

Further Example

Find the remainder when $3x^3 + 0x^2 - 2x + 4$ is divided by $x - 1$.

$$\begin{array}{r}
 3x^2 + 0x + 4 \\
 \underline{-(x - 1)} \\
 3x^2 - 3x + 4 \\
 \underline{-(3x - 3)} \\
 3x + 7 \\
 \underline{-(3x - 3)} \\
 10
 \end{array}$$

The remainder is 10.

For Note: Ensure that any missing terms in the polynomial are filled in, so that the powers decrease by 1 each time.

Test Your Understanding

Find the remainder when 10^{100} is divided by 7.

?

Further Test Your Understanding

Divide by .

?

Exercise 7B

Pearson Pure Mathematics Year 1/AS

Pages 141-142

3 :: The Factor Theorem

$$x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$$

We can see that $x - 2$ is a factor of $x^3 + x^2 - 4x - 4$.
What would happen if x is 2?

?

then $x - 2$ would be a factor.

! The Factor Theorem states that if $p(x)$ is a polynomial then:

- If $p(2) = 0$, then $x - 2$ is a factor of $p(x)$.
- Conversely, if $x - 2$ is a factor of $p(x)$, then $p(2) = 0$.

Examples (in textbook)

Show that $x^2 + 1$ is a factor of $x^4 + 2x^2 + 1$.

?

Fully factorise $x^4 + 2x^2 + 1$.

?

Using Factor Theorem to find unknown coefficients

Given that $x - 2$ is a factor of $x^3 + px^2 + qx + 12$, determine the value of p .

?

Test Your Understanding

Edexcel C2 May 2016
Q2

$$f(x) = 6x^3 + 13x^2 - 4$$

No long in spec.

- (a) ~~Use the remainder theorem to find the remainder when $f(x)$ is divided by $(2x + 3)$. (2)~~
- (b) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$. (2)
- (c) Factorise $f(x)$ completely. (4)

?

Given that $(x + 2)$ is a factor of $f(x)$, determine the value of k .

?

Exercise 7C

Pearson Pure Mathematics Year 1/AS
Pages 145-146

Extensi

1 [MAT 2006 1E] The cubic

Has both x and $x - 1$ as factors.
Determine the values of a and b .

?

2 [MAT 2009 1I] The polynomial

Has $x^2 + 1$ as a factor
A) for no values of a ;
B) for $a = 1$ only;
C) for $a = -1$ only;
D) for $a = 1$ and $a = -1$ only.

?

option B.

The **remainder theorem** states that if $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$. This similarly works whenever $f(x)$ makes the divisor 0.

3 [MAT 2013 1G] Let a be an integer and $f(x)$ be the polynomial

What is the remainder, in terms of a , when $f(x)$ is divided by $(x - 1)$?

?

4 :: Proof

Terminology

A **conjecture** is a mathematical statement that has yet to be proven.

One famous conjecture is **Goldbach's Conjecture**. It states "*Every even integer greater than 2 can be expressed as the sum of two primes.*" It has been verified up to (that's big!); this provides evidence that it is true, but does not prove it is true!

A **theorem** is a mathematical statement that has been proven.

One famous misnomer was **Fermat's Last Theorem**, which states "*If n is an integer where $n > 2$, then $x^n + y^n = z^n$ has no non-zero integer solutions for x, y, z .*" It was 300 years until this was proven in 1995. Only then was the 'Theorem' in the name then correct!

Types of Proof

A proof must show all **assumptions** you are using, have a clear **sequential list of steps** that logically follow, and must cover **all possible cases**.

You should usually make a **concluding statement**, e.g. restating the original conjecture that you have proven.

a. Proof by Deduction

This is the simplest type, where you start from known facts and reach the desired conclusion via deductive steps.

“Prove that the product of two odd numbers is odd.”

?

An **identity** is an equation that is true for **all values** of the variables. e.g. $x^2 - 1 = (x - 1)(x + 1)$ is true only for $x \neq 1$, but $x^2 + 1 = x^2 + 1$ is true for all x .

“Prove that ”

?

Be Warned...

Proof by Deduction requires you to **start from known facts** and end up at the conclusion. It is **not** acceptable to start with to the conclusion, and verify it works, **because you are assuming the thing you are trying to prove.**

Example: Prove that if three consecutive integers are the sides of a right-angled triangle, they must be 3, 4 and 5.

Incorrect Proof:



We are assuming the thing we are trying to prove. We are only assuming things in the 'if' bit. This is fine!

The underlying problem is that this technique doesn't prove there can't be **other** consecutive integers that work – we have only verified 3,4,5 is one such solution.

Correct Proof:



Types of Proof

a. Proof by Deduction

Prove that $x^2 + 2x + 1$ is positive for all values of x .

?

Fro Exam Tip: This is quite a common last part of a question.

Anything squared is at least 0. This is formally known as the '*trivial inequality*'.

Test Your Understanding

Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.

?

which is 2 more than a multiple of 8.

Exercise 7D

Pearson Pure Mathematics Year 1/AS
Pages 149-150

Extension

[STEP 1 2005 Q1] 47231 is a five-digit number whose digits sum to

- (i) Prove that there are 15 five-digit numbers whose digits sum to 43. You should explain your reasoning clearly.
- (ii) How many five-digit numbers are there whose digits sum to 39?

? i

? ii

Other Types of Proof

b. Proof by Exhaustion

This means breaking down the statement into **all possible smaller cases**, where we prove each individual case.

(This technique is sometimes known as 'case analysis')
Prove that n^2 is even for all integers n .

?

c. Disproof by Counter-Example

While to prove a statement is true, we need to prove every possible case (potentially infinitely many!), **we only need one example to disprove** a statement.

This is known as a **counterexample**.
Disprove the statement:
" n^2 is prime for all integers n ."

?

Exercise 7E

Pearson Pure Mathematics Year 1/AS

Pages 152-154
